

## Q: How does a lever work?

By Bill Robertson

**A:** There are two ways of approaching how a lever works. One approach deals with the concept of torque, and the other deals with energy. The torque approach leads to a formula that most people associate with a lever, involving forces and the distance of the force from the fulcrum of the lever. The energy approach helps us understand the relationship between energy input to, and energy output from, a lever. In this column, I'll explain both approaches and the connection between them.

Let's start with the concept of torque. For starters, find yourself a door. Open it so it can move freely. Now try to move the door first by pushing on it near the hinge and second by pushing on it near the edge of the door farthest from the hinge (Figure 1). Much more difficult to move the door near the hinge, yes? You need to apply a larger force there. So, when it comes to causing things to rotate (to be correct, we should talk about *rotational acceleration*, but we'll be okay if we restrict ourselves to objects starting from rest and then rotating), force alone doesn't tell the whole story. There is a quantity known as *torque*, which is the applied force multiplied by its *moment arm*. For a force applied at a right angle to the rotating object, the moment arm is the distance from the place where the force is applied to the point about which the object is rotating. Figure 2



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"It's great, Heidi! All the privacy I want without locking the door ... and they probably won't figure it out until I'm in college!"

illustrates the moment arms for the two forces applied to the door. The larger the moment arm, the less force is necessary to get the door moving. So, the torque from an applied force is defined as

$$\text{Torque} = (\text{Applied Force}) \times (\text{Moment Arm})$$

Let's apply this to a lever, shown in Figure 3, page 60. This lever is a ruler, using a pencil as a fulcrum. Feel free to get your own ruler and pencil to play along and experience the operation of a lever. This lever is being used to lift a rock.

Now consider a point where you are using the lever to lift the rock, and nothing is moving. If there is no motion, we know that all the forces acting on the lever must cancel out (an unbalanced force would cause the lever to accelerate, or change its motion). We also know that all of the torques acting on the lever must cancel out (an unbalanced torque would cause the lever to have a rotational acceleration, meaning it would speed up or slow down its rotational speed). The fulcrum of the lever is the point of rotation. The torque you apply to the lever tends to make it rotate in one direction, whereas the torque the rock

Figure 1a.



Figure 1b.



applies to the lever tends to make the lever rotate in the opposite direction. These torques must cancel. So, the force you apply times its moment arm equals the force the rock applies times its moment arm. (The force the lever applies to the rock is equal to the force the rock applies to the lever. This is an application of Newton's third law.) See Figure 4, page 60 and the following equation:

$$\begin{array}{l} \text{Torque you} \\ \text{apply} \end{array} = \begin{array}{l} \text{Torque} \\ \text{rock applies} \end{array}$$

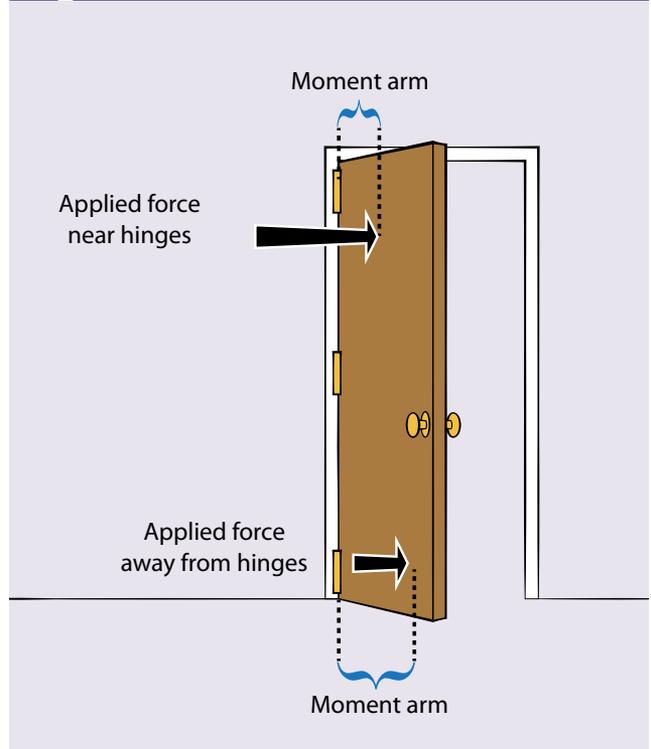
$$(F_1)(D_1) = (F_2)(D_2)$$

If you've ever learned any formulas associated with levers, this is most likely the one. When I was a kid, it was known as the "Law of the Lever." In our example of lifting the rock,  $D_1$  is longer than  $D_2$ . Therefore, for things to equal out,  $F_1$  is smaller than  $F_2$ . See Figure 5, page 61. The

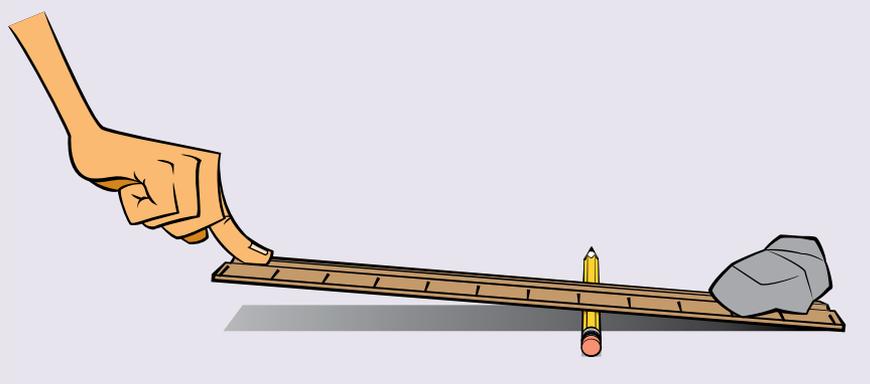
rock experiences a larger force than the one you apply, which is why levers are useful machines. (Again, remember that the force applied to the rock by the lever is equal to the force applied to the lever by the rock.) If you're playing along with a ruler, a pencil, and a rock, you can experience how easy it is to lift the rock in this setup.

The equation, although it tells us how forces and moment arms

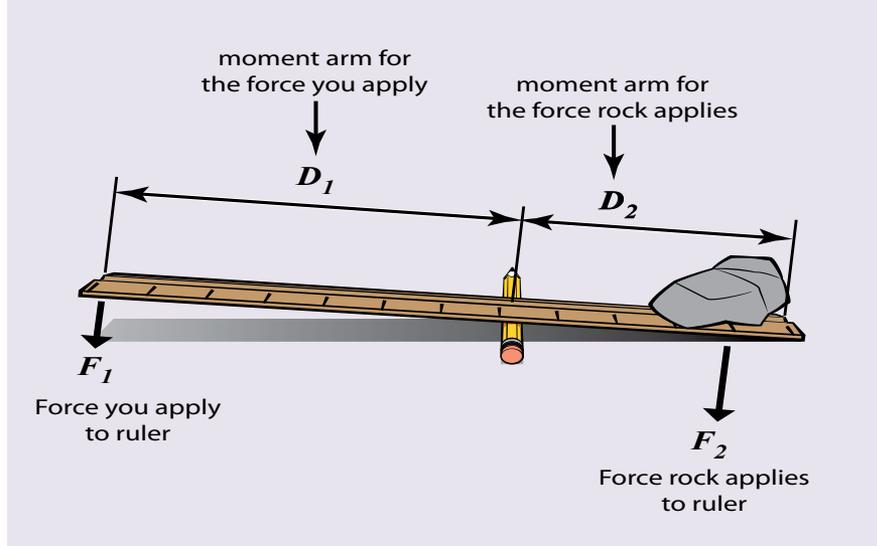
Figure 2.



**Figure 3.**



**Figure 4.**



compare and that we can end up with a force larger (or smaller, depending on the location of the fulcrum) than the one we apply, doesn't say anything about input energy versus output energy. I could use the equation to get directly to energy concepts, but that approach might not be clear. Instead, I'll do something a bit different.

## Work It Out

Central to many ideas in energy is the concept of *work*. The special definition of the work done on an object that scientists use is the following:

$$\text{Work} = (\text{applied force}) \times (\text{distance object moves in the direction of the force})$$

When you do work on an object, you change its *kinetic* energy. Work is also involved in the concept of potential energy. In analyzing how a lever operates, though, we don't have to deal with the concepts of kinetic and potential energy. All we have to do is look at the work you are doing on the lever and compare it to the work done on the rock. The work you do on the lever gives the lever energy, whereas the work the lever does on the rock represents energy that exits the lever. If you buy into the idea that doing work on something is the equivalent of giving it energy, then we are talking about energy input to the lever and energy output from the lever. Anyway, Figure 6, p. 62, shows the forces and distances involved in the work in and work out from the lever.

Notice that the forces involved are the same as the forces we used when talking about torques, but the distances are different. Now, if we assume that there's no energy "loss" in our lever (e.g., through friction that leads to energy leaving as heat), then the energy in is equal to the energy out and the work in is equal to the work out:

$$(\text{Work input to lever}) = (\text{Work output from lever})$$

$$(F_1)(d_1) = (F_2)(d_2)$$

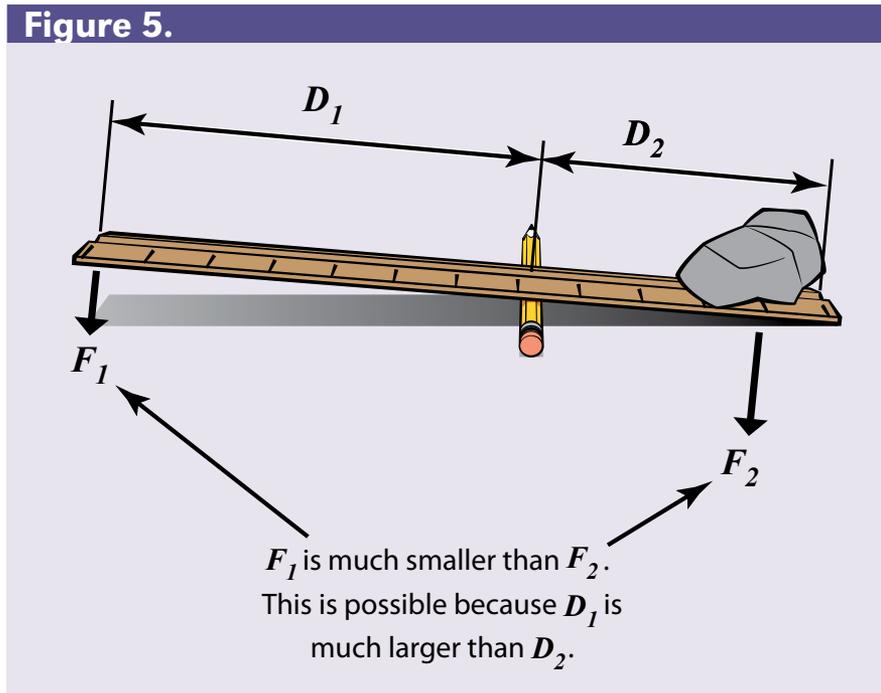
Here I'm using *d* for distance to distinguish from the *D* used previously. They're both distances, but they aren't the same distances. *D* is the distance from the applied force to the fulcrum and the *d* is the distance the end of the lever moves.

Notice that, just as  $D_1$  is greater

than  $D_2$  (Figure 4),  $d_1$  is greater than  $d_2$  (Figure 6, p. 62). So, the two force and distance equations, involving different  $ds$ , give us the same information. Both tell us that, because of the difference in the distances,  $F_1$  is less than  $F_2$ . Whereas the equation involving capital  $D$ s simply tells us that the forces are different, the equation involving lowercase  $d$ s tells us something about energy. Even though the force applied to the rock is larger than the force we apply to the lever, there is no gain in energy in the process. Even if there are no losses due to friction (and there are!), the energy input (or work input) is equal to the energy output (or work output). You see, there's a trade-off. You get a larger force moving the rock than you exert on the end of the lever, but you have to move your end of the lever farther than the rock moves. You're trading force for distance.

This trade-off of force and distance is common to most simple machines. If you've ever used a pulley system to lift an object, you know you can lift objects much heavier than you would be able to lift directly. But you have to pull on your end of the rope a lot (lots of distance) to get the heavy object to move even a little bit. The jack for your car is also a simple machine. You have to pump or turn the jack handle many times to get the car to lift a short distance.

In coming up with the earlier equation comparing force and distance, I assumed that no energy was lost through friction. But in any real lever, there will be friction. So, the work the lever does on the rock is actually less than the work you do on the lever. Not only can you not get



something for nothing (more work out than you put in), you get less something out than you put in.

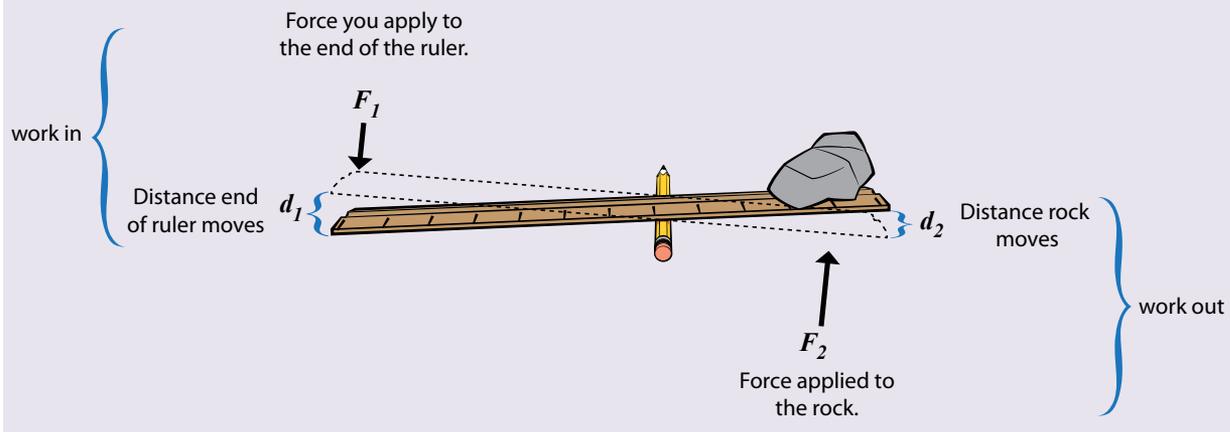
### Everyday Levers

I should mention the connection between the torque equation for the lever and the energy equation for the lever. Although the distances are different, for small motions of the lever those distances are exactly proportional. If you multiply the distance  $D_1$  by the size of the angle the lever makes with the surface, you get  $d_1$  (for small movements of the lever). If I had multiplied the first equation by that angle, I would have gone directly from an analysis with torques to an analysis with energy. I didn't want to do that, because the connection wouldn't have

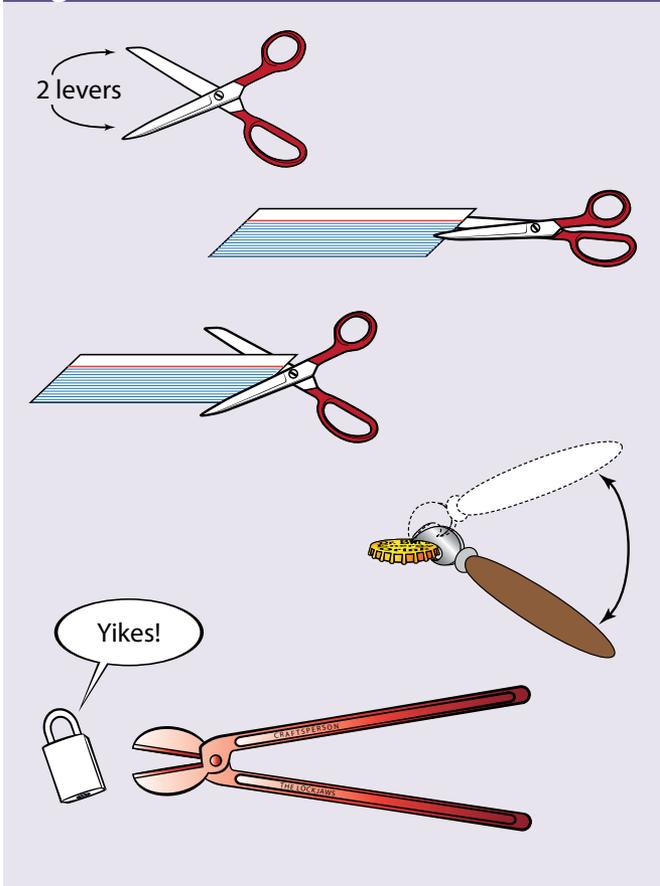
been obvious. Anyway, each equation will give you the same answer for the relative sizes of  $F_1$  and  $F_2$ . And that's reassuring. We wouldn't want our use of different equations to change reality!

To end the column, let's look at a few common levers, shown in Figure 7, page 62. Ever try to remove a non-twist-off bottle cap with your hands? Difficult. It's easy with a bottle opener, though. You move the handle a large distance with a small force, and the opener applies a large force to the bottle cap, moving it just a short distance. Scissors are a combination of two levers, joined at the hinge. If you try to cut an index card using the end of the scissors, it's difficult. That's because in that situation, the moment arm for the force you apply is about the same as the

**Figure 6.**



**Figure 7.**



moment arm for the force applied to the index card. If the index card is closer to the hinge of the scissors, though, the cutting is easy. Large moment arm for the force you apply and small moment arm for the force applied to the card. That means a much greater force applied to the card than you apply to the handle. This force advantage gets larger if the difference between moment arms is larger, as in the use of lock cutters (Figure 7).

In all of these examples of everyday levers, you can increase the force applied to an object, but there is no gain in energy. The work you get out of the lever (or levers) is no larger than the work you put into it. You are trading distance (you move your end of the lever a large distance and the other end moves little) for force (much larger force applied to the object than you exert on your end).

There's a famous quote from Archimedes that goes something like, "Give me a lever long enough and a fulcrum on which to place it, and I shall move the world." Is that possible? Well yes, but we'd have to dismantle a fair portion of the Earth to get materials for the lever. Plus, the fulcrum would have to be outside the Earth—maybe one of those turtles that are supposed to be holding up the Earth according to ancient myth. ■

*Bill Robertson (wrobert9@ix.net com.com) is the author of the NSTA Press book series, Stop Faking It! Finally Understanding Science So You Can Teach It.*